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AVERAGE AND PROBABILITY.

125. Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

A circle with unknown radius is described with its center at the extremity of the major axis of a given ellipse. Required, the average area common to the circle and the ellipse.

Solution by the PROPOSER.

Take for equations of the ellipse and variable circle, respectively,

$$\frac{(x-a)^2}{a^2} + \frac{y^2}{b^2} = 1 \dots (1), \text{ and } x^2 + y^2 = r^2 \dots (2).$$

From (1), (2), and the properties of the ellipse with the usual notation, we find, after reducing,

$$r = 2a \sin \frac{1}{2} \theta \sqrt{1 - \varepsilon^2 \cos^2 \frac{1}{2} \theta} \dots (3),$$

and $a = 2 \tan^{-1} \left(\frac{b \cot \frac{1}{2} \theta}{a} \right)$, the angle formed by the radii drawn to the points of intersection of the ellipse and circle. Then the area common to both is

$$S = \int a r d r = 4a^2 \sin^2 \frac{1}{2} \theta (1 - \varepsilon^2 \cos^2 \frac{1}{2} \theta) \tan^{-1} \left(\frac{b \cot \frac{1}{2} \theta}{a} \right) + ab \theta - ab \sin \theta \dots (4).$$

Hence the required average area is

$$A = \frac{1}{2a} \int_0^{2a} S dr = \frac{1}{2a} \left[rS - \int r dS \right]_0^{2a} \dots (5).$$

Now substituting in (5) the value of r from (3), the values of S and dS from (4), integrating, and observing that when $r=0$, $\theta=0$; and when $r=2a$, $\theta=\pi$, we finally have

$$\begin{aligned} A = & \frac{1}{2a} \left[\frac{8}{3} a^3 \sin^3 \frac{1}{2} \theta [1 - \varepsilon^2 \cos^2 \frac{1}{2} \theta]^{\frac{1}{2}} \tan^{-1} \left(\frac{b \cot \frac{1}{2} \theta}{a} \right) + 2a^2 b \theta \sin \frac{1}{2} \theta [1 - \varepsilon^2 \cos^2 \frac{1}{2} \theta]^{\frac{1}{2}} \right. \\ & \left. - 2a^2 b \sin \theta \sin \frac{1}{2} \theta [1 - \varepsilon^2 \cos^2 \frac{1}{2} \theta]^{\frac{1}{2}} \right. \\ & \left. - \frac{2a^2 b}{3\varepsilon^2} [4\varepsilon^2 - 1] \cos^{-1} [\varepsilon \cos \frac{1}{2} \theta] + \frac{2a^2 b}{3\varepsilon^2} [4\varepsilon^2 + 1] \cos \frac{1}{2} \theta [1 - \varepsilon^2 \cos^2 \frac{1}{2} \theta]^{\frac{1}{2}} \right. \\ & \left. - \frac{4a^2 b}{3} \cos^3 \frac{1}{2} \theta [1 - \varepsilon^2 \cos^2 \frac{1}{2} \theta]^{\frac{1}{2}} \right]_0^\pi = ab \left[\pi - \left(\frac{4\varepsilon^2 - 1}{3\varepsilon^3} \right) \sin^{-1} \varepsilon - \frac{b}{a} \left(\frac{2\varepsilon^2 + 1}{3\varepsilon^2} \right) \right]. \end{aligned}$$

Also solved by G. B. M. ZERR. If space permits his solution will appear in the next issue.